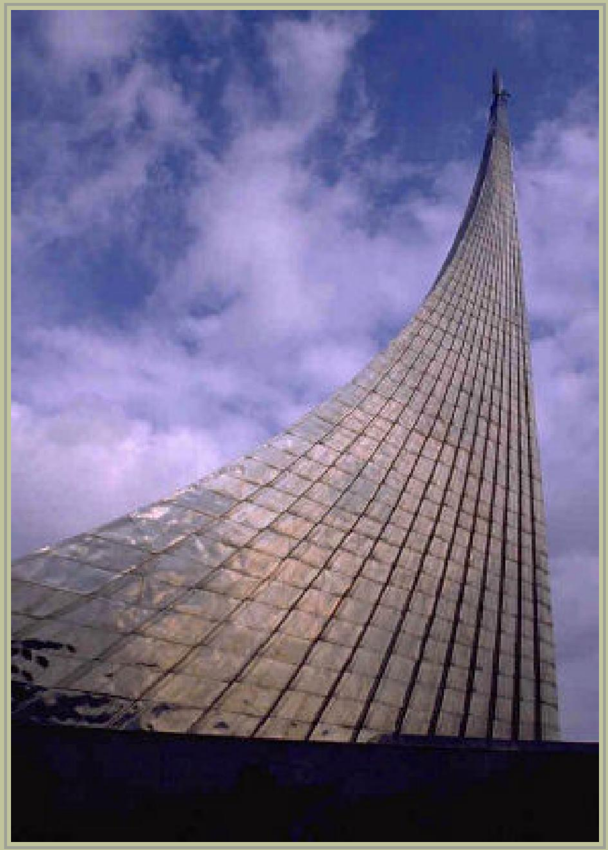

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Variance Decomposition of REIT Returns

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June 16, 2008

Introduction

- Two components to REIT returns:
  1. Pure real estate
  2. Stock market
- Focus on stock market component
- Objective:
  1. What factors affect REIT Returns
  2. Returns can be broken down into two components: Expected and Unexpected
  3. Estimate model using Classical and Bayesian methodologies and contrast results

Campbell (1991) VAR

- Campbell (1991) VAR based on a dynamic dividend growth model
- Follows on Campbell-Shiller (1988) log linear approximation
- Breaks down postwar US stock market data into expected and unexpected return components
- Expected Components include:
  1. Dividend Price Ratio
  2. Relative Treasury Bill Rate
- Unexpected Components are corresponding residuals in VAR

REIT Pricing

- Early studies: little or no connection between REITs and physical properties
- Myer, Seiler and Webb (2001): REITs and publicly held real estate are no substitutes for each other
  BUT...
- Giliberto (1990): Once stock and bond market factors are accounted for, positive relationship between REITs and privately held real estate
- Barkham and Geltner (1995): Strong co-movement between NAREIT and NCREIF returns

Variance Decomposition of REITs

- Clayton and MacKinnon (2003):
  - Use a variance decomposition to study the relationship between REITs, financial assets and physical real estate
  - Structural break in REIT returns:
    1. Over 70s and 80s REIT returns strongly related to large cap stocks
    2. Since 90s, closer association to small cap and physical real estate
- Consensus: REITs hybrid of stocks, bonds and physical real estate

Bayesian Advantages

- Exact results; not based on asymptotic normality
- Bias in VAR systems can be substantial and cannot be easily adjusted (Stambaugh (1999))
- Possibility of conditioning on different information set thereby allowing for different investor beliefs
- Imposition of stationarity is almost trivial

Dynamic Dividend Growth Model

- If p_t denotes the logarithm of the price and d_t denotes the dividend, then:

$$r_{t+1} = p_{t+1} - p_t + \ln(1 + \exp(d_{t+1} - p_{t+1})) \tag{1}$$

- Applying a first order Taylor expansion around the mean log dividend price ratio, we have:

$$r_{t+1} \approx h + \rho p_{t+1} + (1 - \rho) d_{t+1} - p_t \tag{2}$$

where ρ is the discount factor

$$\rho \equiv \frac{1}{1 + \exp\overline{(d - p)}}; \quad h \equiv -\ln(\rho) - (1 - \rho)\ln(1/\rho - 1) \tag{3}$$

Excess Returns

- Taking conditional expectations from equation (2) we have:

$$E(p_t) = \frac{h}{1-\rho} + E_t\left[\sum_{i=0}^{\infty}\rho^i\left((1-\rho)d_{t+1+i} - r_{t+1+i}\right)\right] \quad (4)$$

- Substituting (4) back in (2) and if e_t denotes the excess returns:

$$e_{t-1} - e_t = (E_{t-1} - E_t) \times \zeta \quad (5)$$

where ζ is:

$$\zeta = \left[\sum_{i=0}^{\infty}\rho^i \triangle d_{t+1+i} - \sum_{i=0}^{\infty}\rho^i r^f_{t+1+i} - \sum_{i=1}^{\infty}\rho^i e_{t+1+i}\right] \quad (6)$$

VAR Set Up

- Equation (6) can be broken down into:
  1. News about future dividends
  2. News about bond markets
  3. News about future excess returns
- Equation (6) is not directly observable
- Use a VAR to obtain empirical counterparts

$$z_t = \beta z_{t-1} + u_t \tag{7}$$

- where z_t is the vector of stationary variables:

$$z_t = [e_t, dy_t, b_t, x_t] \tag{8}$$

News Components

- If e_1 and e_2 are defined as the first two columns on an identity matrix, "news about future returns" can be written as:

$$\begin{aligned} \tilde{e}_{e,t+1} &\equiv (E_{t+1} - E_t) \sum_{i=1}^{\infty} \rho^i e_{t+1+i} \\ &= e_1' \rho\beta (I - \rho\beta)^{-1} u_{t+1} \end{aligned} \tag{9}$$

- Similarly "news about interest rates" can be given by:

$$\begin{aligned} \tilde{e}_{r,t+1} &\equiv (E_{t+1} - E_t) \sum_{i=1}^{\infty} \rho^i r^f_{t+1+i} \\ &= e_2' \rho\beta (I - \rho\beta)^{-1} u_{t+1} \end{aligned} \tag{10}$$

Bayesian Approach

- Basic idea is to extract maximum possible information from existing data ("prior") to form expectations about the future ("posterior")
- Concept is similar to Bayes rule
- Uses Markov Chain Monte Carlo Methods for estimation
- Various priors can be used depending on the information set available

Prior Specifications

- Prior used in this paper: Normal-Wishart Prior
- Normal-Wishart Prior:

$$\pi(\beta) = f_N^{k^2}\left(b_0, \Omega_0^{-1}\right) \quad (11)$$

$$\pi(\Sigma^{-1}) = g_w^k\left(\nu_0, S_0^{-1}\right) \quad (12)$$

- where (11) and (12) denote:
  - the p.d.f. of the multivariate Normal distribution
  - the k-dimensional Wishart p.d.f with ν_0 being the degree of freedom parameter and S_0 being the inverse of the covariance matrix
  - Stationarity is imposed by adding the following constraint:

$$\beta^* \equiv \{\beta \text{ such that } \max|\text{eig}(\beta)| < 1\} \quad (13)$$

Posterior Simulations

- The corresponding posteriors are given by:

$$
\begin{aligned}
p(\beta|\Sigma, X, y) &= f_N^{k^2}(\tilde{\beta}, \tilde{\Omega}^{-1}) && (14)\\
p(\Sigma^{-1}|\beta, X, y) &= g_w^k(\tilde{\nu}, \tilde{S}^{-1}) && (15)
\end{aligned}
$$

- where

$$
\begin{aligned}
\tilde{\nu} &= T + \nu_0 && (16)\\
\varepsilon_t &= y_t - X_t\beta && (17)\\
\tilde{S} &= S_0 + \varepsilon_t^{'}\varepsilon_t && (18)\\
\tilde{\Omega} &= \tilde{X}'\tilde{X} + b_0 && (19)\\
\tilde{\beta} &= \tilde{\Omega}^{-1}(\tilde{X}'\tilde{y} + \Omega_0 b_0) && (20)\\
\tilde{y}_t &= C^{-1}y_t && (21)
\end{aligned}
$$

Gibbs Sampler

- If $\theta = (\beta, \Sigma)$, then the Gibbs sampler works as follows:
  1. set $i = 0$ and initialize $\theta^{(i)} = \theta_0$.
  2. sample from

$$\begin{aligned}
\theta_1^{(i+1)} &\sim p\left(\theta_1 | \theta_2^{(i)}, \ldots, \theta_k^{(i)}, y\right) \\
\theta_2^{(i+1)} &\sim p\left(\theta_2 | \theta_1^{(i+1)}, \theta_3^{(i)}, \ldots, \theta_k^{(i)}, y\right) \\
&\vdots \\
\theta_j^{(i+1)} &\sim p\left(\theta_j | \theta_1^{(i+1)}, \ldots, \theta_{j-1}^{(i+1)}, \theta_{j+1}^{(i)}, \ldots, \theta_k^{(i)}, y\right) \\
&\vdots \\
\theta_k^{(i+1)} &\sim p\left(\theta_k | \theta_1^{(i+1)}, \ldots, \theta_{k-1}^{(i+1)}, y\right)
\end{aligned}$$

  3. set $i = i + 1$

Summary Statistics

Statistic	e_t	dy_t	b_t	x_t
Number of Obs., N		432		
Mean, μ	-0.0005	0.0097	0.0831	0.0012
Std. Dev., σ	0.0118	1.1743	0.0217	0.0454
Skewness, S	-0.5424	-0.2981	1.9769	-0.5527
Kurtosis, K	7.4869	2.8179	7.0335	5.1699
JB Stat.	**383.6**	**7.0**	**376.3**	**106.7**
ARCH-LM Stat.	**6.05**	**388.01**	**364.53**	**4.19**
	{0.0139}	**{0.0000}**	**{0.0000}**	**{0.0407}**

Classical Estimation

	e_t	dy_t	b_t	x_t	R^2
	Panel A: 197201 - 200712				
e_{t+1}	0.0029	0.0008	0.0054	0.1543	0.0422
	(0.0037)	(0.0108)	(0.0359)	(0.0035)	
$\vdots$			$\vdots$		
	Panel B: 197201 - 198912				
e_{t+1}	0.0078	-0.0002	0.0079	0.2628	0.0430
	(0.0097)	(0.0280)	(0.0930)	(0.0094)	
$\vdots$			$\vdots$		
	Panel C: 198912 - 200712				
e_{t+1}	0.0207	-0.0021	0.0039	0.0491	0.0505
	(0.0058)	(0.0166)	(0.0558)	(0.0051)	
$\vdots$			$\vdots$		

Bayesian Estimation

	e_t	dy_t	b_t	x_t	R^2
	Panel A: 197201 - 200712				
e_{t+1}	-0.0115	-0.0139	0.1923	0.0079	0.0520
	(0.0629)	(0.0064)	(0.0000)	(0.0000)	
⋮			⋮		
	Panel B: 197201 - 198912				
e_{t+1}	-0.0302	0.0497	0.0104	0.0577	0.0212
	(0.1027)	(0.0111)	(0.0000)	(0.0000)	
⋮			⋮		
	Panel C: 198912 - 200712				
e_{t+1}	0.0075	-0.0164	-0.0478	0.0660	0.0486
	(0.0782)	(0.0046)	(0.0000)	(0.0000)	
⋮			⋮		

Estimation Summary

- 10,000 MCMC iterations carried out with first 1,000 regarded as "burn-ins"
- Structural break in NAREIT returns over period considered
- Confirmed by Chow Statistic
- Bayesian and Classical Results differ
- Higher degree of confidence in Bayesian Results

Variance Decomposition

Classical	197201-200712	197201-198912	1990-200712
η_e	0.0331	0.0765	0.0076
η_r	75.93	110.03	40.4520
η_d	96.62	136.93	55.1797
$2cov(\tilde{e}_e, \tilde{e}_r)$	-171.27	-245.45	-94.8403
$-2cov(\tilde{e}_e, \tilde{e}_r)$	-3.18	-6.0372	-1.359
$-2cov(\tilde{e}_d, \tilde{e}_r)$	2.85	5.4462	0.9770
MCMC			
η_e	0.0520	0.0028	0.0037
η_r	1.6068	3.0310	2.4905
η_d	4.4829	7.5001	6.4644
$2cov(\tilde{e}_e, \tilde{e}_r)$	-4.8484	-9.4931	-7.9953
$-2cov(\tilde{e}_e, \tilde{e}_r)$	-0.8706	-0.1378	0.1044
$-2cov(\tilde{e}_d, \tilde{e}_r)$	0.5775	0.0969	-0.0678

Conclusion

- Variance Decomposition under Classical and Bayesian Estimation carried out
- Difference between Classical and Bayesian Coefficient estimates
- "News about Dividends" account for most of the variance seen
- Future research: ex. post vs. ex. ante behaviour